解析　第１回小テスト問題

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次の関数を微分せよ.

(1) $y = x^5 + 5x^3 + 15x$

$$y' = 5x^4 + 15x^2 + 15 = 5(x^4 + 3x^2 + 3)$$

(2) $y = x^7 - \dfrac{1}{x^7}$

$$y' = 7x^6 - \{x^{-7}\}' = 7x^6 + 7x^{-8} = 7(x^6 + \frac{1}{x^8})$$

(3) $y = \dfrac{x^2 - 1}{x^2 + 1}$

$$y' = \frac{\{x^2-1\}'(x^2+1) - (x^2-1)\{x^2+1\}'}{(x^2+1)^2} = \frac{2x \cdot (x^2+1) - (x^2-1) \cdot 2x}{(x^2+1)^2}$$
$$= \frac{4x}{(x^2+1)^2}$$

(4) $y = x\cos x + \sin x$

$$y' = \{x\}'\cos x + x\{\cos x\}' + \cos x = \cos x - x\sin x + \cos x = 2\cos x - x\sin x$$

(5) $y = e^x \sin x$

$$y' = \{e^x\}'\sin x + e^x\{\sin x\}' = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$$

(6) $y = \log(1 + x^2)$

$$y' = \frac{1}{1+x^2} \cdot \{1 + x^2\}' = \frac{2x}{1+x^2}$$

(7) $y = x\log 2x - x$

$$y' = \{x\}'\log 2x + x\{\log 2x\}' - 1 = \log 2x + x \cdot \frac{1}{2x} \cdot \{2x\}' - 1 = \log 2x$$

(8) $y = \dfrac{\log x}{x}$

$$y' = \frac{\{\log x\}' \cdot x - \log x \cdot \{x\}'}{x^2} = \frac{\frac{1}{x} \cdot x - \log x}{x^2} = \frac{1 - \log x}{x^2}$$

(9) $y = (x^2 + 2x + 5)^{100}$

$$y' = 100(x^2 + 2x + 5)^{99}\{x^2 + 2x + 5\}' = 100(x^2 + 2x + 5)^{99}(2x + 2)$$

$$= 200(x + 1)(x^2 + 2x + 5)^{99}$$

(10) $y = \dfrac{1}{(x^2 + 2x + 1)^{10}}$

$$y' = \{(x + 1)^{-20}\}' = -20(x + 1)^{-21} \cdot \{x + 1\}' = -20(x + 1)^{-21}$$

(11) $y = \sqrt{x^2 + 2x + 5}$

$$y' = \{(x^2 + 2x + 5)^{\frac{1}{2}}\}' = \frac{1}{2}(x^2 + 2x + 5)^{-\frac{1}{2}} \cdot \{x^2 + 2x + 5\}'$$

$$= \frac{1}{2}(x^2 + 2x + 5)^{-\frac{1}{2}} \cdot (2x + 2) = \frac{x + 1}{\sqrt{x^2 + 2x + 5}}$$

(12) $y = \log(\log(x^2 + 1))$

$$y' = \frac{1}{\log(x^2 + 1)} \cdot \{\log(x^2 + 1)\}' = \frac{1}{\log(x^2 + 1)} \cdot \frac{1}{x^2 + 1} \cdot \{x^2 + 1\}'$$

$$= \frac{2x}{(x^2 + 1)\log(x^2 + 1)}$$

(13) $y = e^{x^2}$

$$y' = e^{x^2} \cdot \{x^2\}' = 2xe^{x^2}$$

(14) $y = \sin^{-1}\dfrac{x}{3}$

$$y' = \frac{1}{\sqrt{1 - (\frac{x}{3})^2}} \cdot \left\{\frac{x}{3}\right\}' = \frac{1}{\sqrt{1 - (\frac{x}{3})^2}} \cdot \frac{1}{3} = \frac{1}{\sqrt{9 - x^2}}$$

(15) $y = \tan^{-1} x + \tan^{-1}\dfrac{1}{x}$

$$y' = \frac{1}{1 + x^2} + \frac{1}{1 + (\frac{1}{x})^2} \cdot \left\{\frac{1}{x}\right\}' = \frac{1}{1 + x^2} + \frac{1}{1 + (\frac{1}{x})^2} \cdot (-\frac{1}{x^2})$$

$$= \frac{1}{1 + x^2} - \frac{1}{x^2 + 1} = 0$$